

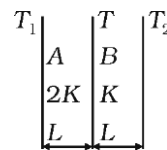
Previous Years Questions	JEE Advanced/Solutions
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1.(C)

2.(B) $H = \frac{2k(T_1 - T)}{L} = \frac{kA(T - T_2)}{L} \Rightarrow 2T_1 - 3T = -T_2$

Adding T_1 on both sides:

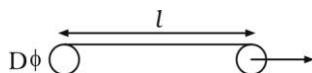
$$3T_1 - 3T = T_1 - T_2 \Rightarrow T_1 - T = \frac{T_1 - T_2}{3} = \frac{36}{3} = 12^\circ\text{C}$$



3.(A) $\frac{T}{\pi D^2} = Y \frac{\Delta l}{l}$

$$\Delta l = \frac{4\pi l}{\pi Y D^2}$$

$$\Delta l \propto \frac{l}{D^2} \Rightarrow \Delta l \text{ is maximum for A.}$$



4.(A) Heat required, $Q = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15) = 72800 \text{ cal}$

Let m gram of steam is condensed. Then heat loss $= m \times 540 + m \times 1 \times 20 = 560m$

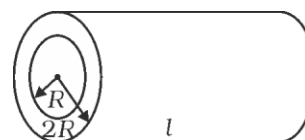
Heat loss = heat gain

$$\Rightarrow 560m = 72800 \Rightarrow m = 130g = 0.130kg$$

5.(C) Let R_1 and R_2 be the thermal resistance of inner and outer portions since temperature difference at both ends is same, the resistances are in parallel. Hence,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K(4\pi R^2)}{l} = \frac{K_1(\pi R^2)}{l} + \frac{K_2(3\pi R^2)}{l} \Rightarrow K = \frac{3K_2 + K_1}{4}$$



6.(C) Thermal stress $\sigma = Y\alpha\Delta\theta$

But $\sigma_1 = \sigma_2 \Rightarrow Y_1\alpha_1\Delta\theta = Y_2\alpha_2\Delta\theta \Rightarrow \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$

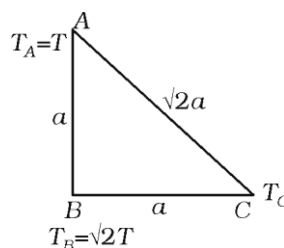
7.(B) $\frac{dQ}{dt} = P = KA \frac{\Delta\theta}{\Delta x} = K \cdot 4\pi \frac{R^2 T}{\Delta x}$ (as mean radius is R) $\Rightarrow$ Thickness $(\Delta x) = \frac{4\pi R^2 KT}{P}$

8.(B) Heat lost $= ML = Pt$

9.(B) $\left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CA}$ (for T_c to be constant)

$$\Rightarrow \frac{kA(\sqrt{2}T - T_c)}{a} = \frac{kA(T_c - T)}{\sqrt{2}a}$$

Solve to get: $\frac{T_c}{T} = \frac{3}{\sqrt{2} + 1}$



10.(D) According to Stefan's law $\Delta Q = \epsilon\sigma AT^4\Delta t$

Also $\Delta Q = mc\Delta T \Rightarrow \Delta Q = mc\Delta T = \epsilon\sigma AT^4\Delta t$

$$\Rightarrow \frac{\Delta T}{\Delta t} = \frac{\epsilon\sigma AT^4}{mc} = \frac{\epsilon\sigma T^4}{mc} \left[\pi \left(\frac{3m}{4\pi\rho} \right)^{2/3} \right] = k \left(\frac{1}{m} \right)^{1/3} \therefore \frac{\Delta T_1 / \Delta t_1}{\Delta T_2 / \Delta t_2} = \left(\frac{m_2}{m_1} \right)^{1/3} = \left(\frac{1}{3} \right)^{1/3}$$

- 11.(B)** According to Wein's displacement law, $\lambda_m T = \text{constant}$ where λ_m is the wavelength for which intensity of radiation emitted is maximum

$$(\lambda_m)_S T_S = (\lambda_m)_{NS} \times T_{NS}$$

S = Sun

$$NS = \text{north star} \quad \Rightarrow \quad \frac{T_S}{T_{NS}} = \frac{(\lambda_m)_{NS}}{(\lambda_m)_S} = \frac{350}{510} = 0.69$$

- 12.(D)** The energy radiated per second by a black body is given by Stefan's law,

$$\frac{E}{t} = \sigma T^4 \times A$$

Where A is the surface area of the black body

$$\frac{E}{t} = \sigma T^4 \times 4\pi r^2$$

Since black body is sphere, $A = 4\pi r^2$

Case (i) $\frac{E}{t} = 450$, $T = 500K$, $r = 0.12m$,

$$\therefore 450 = 4\pi\sigma(500)^4(0.12)^2 \quad \dots (i)$$

Case (ii) $\frac{E}{t} = ?$ $T = 1000K$, $r = 0.06m$ $\dots (ii)$

Dividing Equation. (ii) by Equation.(i), we get :

$$\frac{E/t}{450} = \frac{(1000)^4(0.06)^2}{(500)^4(0.12)^2} = \frac{2^4}{2^2} = 4 \quad \Rightarrow \quad \frac{E}{t} = 450 \times 4 = 1800W$$

- 13.(D)** Wien's displacement law is

$$\lambda_m T = b \quad (b = \text{Wien's constant})$$

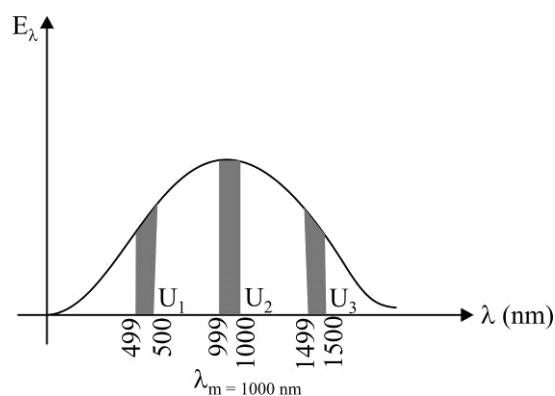
$$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880 \text{ K}}$$

$$\lambda = 1000 \text{ nm}$$

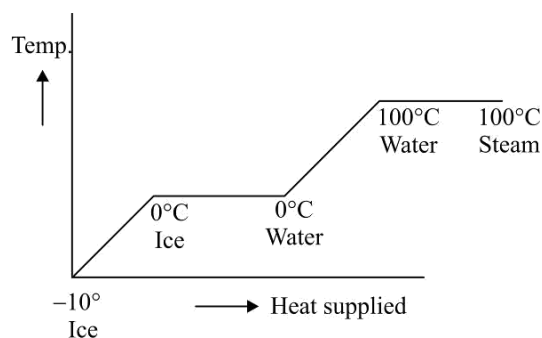
Energy distribution with wavelength will be as follows:

From the graph it is clear that

$$U_2 > U_1 \quad (\text{in fact } U_2 \text{ is maximum})$$



- 14.(A)**



15.(A) $k_1 = \left(\frac{\rho_{Fe}}{\rho_{Hg}} \right)_{0^\circ C}$ and $k_2 = \left(\frac{\rho_{Fe}}{\rho_{Hg}} \right)_{60^\circ C}$

Here, ρ = Density

$$\therefore \frac{k_1}{k_2} = \frac{(\rho_{Fe})_{0^\circ C}}{(\rho_{Hg})_{0^\circ C}} \times \left(\frac{\rho_{Hg}}{\rho_{Fe}} \right)_{60^\circ C} = \frac{(1 + 60\gamma_{Fe})}{(1 + 60\gamma_{Hg})}$$

16.(C) The length of each rod increases by the same amount

$$\therefore \Delta l_a = \Delta l_s \Rightarrow l_1 \alpha_a t = l_2 \alpha_s t$$

$$\Rightarrow \frac{l_2}{l_1} = \frac{\alpha_a}{\alpha_s} \Rightarrow \frac{l_2}{l_1} + 1 = \frac{\alpha_a}{\alpha_s} + 1 \Rightarrow \frac{l_2 + l_1}{l_1} = \frac{\alpha_a + \alpha_s}{\alpha_s} \Rightarrow \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

17.(B) Heat required to convert 5 kg of water at $20^\circ C$ to 5 kg of water at $0^\circ C = mC_w \Delta T = 5 \times 1 \times 20 = 100 \text{ kcal}$.

Heat released by 2 kg ice at $-20^\circ C$ to convert 2 kg of ice at $0^\circ C = mC_{ice} \Delta T = 2 \times 0.5 \times 20 = 20 \text{ kcal}$

How much ice at $0^\circ C$ will convert into water at $0^\circ C$ for giving another 80 kcal of heat $Q = mL$

$$\Rightarrow 80 = m \times 80 \Rightarrow m = 1 \text{ kg}$$

Therefore, the amount of water at $0^\circ C = 5 \text{ kg} + 1 \text{ kg} = 6 \text{ kg}$

Thus, at equilibrium we have (6 kg water at $0^\circ C$ + 1 kg ice at $0^\circ C$)

18.(C) $Y = \frac{Fl}{A\Delta l} = \frac{20 \times 1}{10^{-6} \times 1 \times 10^{-4}} = 20 \times 10^{10} = 2 \times 10^{11} \text{ N/m}^2$

19.(C) Temperature of liquid oxygen will first increase till boiling point.

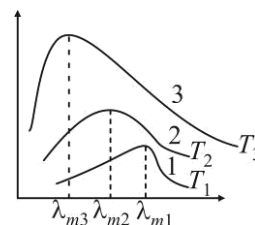
Then phase change will take place from liquid to gas during which temperature remains same. After this temperature of the gaseous oxygen will further increase. All this is correctly shown by graph (c).

20.(A) According to Wien's displacement law

$$\lambda_m \times T = \text{constant}$$

Here $\lambda_{m3} < \lambda_{m2} < \lambda_{m1} \Rightarrow T_3 > T_2 > T_1$

The temperature of the sun is higher than that of the welding arc which in turn is greater than the tungsten filament (A) is the correct option.



21.(C) Heat radiated at temperature T : $Q = \epsilon \sigma AT^4 = 0.6 \sigma AT^4$

22.(A) One calorie is the heat required to raise the temperature of 1 g of water from $14.5^\circ C$ to $15.5^\circ C$ at 760 mm of Hg. (i.e. at 1 atm)

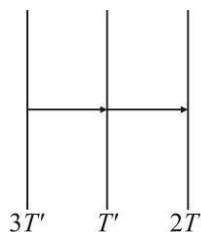
23.(D)

24.(C) In steady state, energy absorbed per second by middle plate is equal to energy released per second by middle plate.

$$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$$

$$(3T)^4 - (T')^4 = (T')^4 - (2T)^4$$

$$2(T')^4 = (16 + 81)T^4, \quad T' = \left(\frac{97}{2} \right)^{1/4} T$$



25.(A) $R_1 = \frac{L}{kA} + \frac{L}{2kA} = \frac{3L}{2kA}$, $\frac{1}{R_2} = \left(\frac{L}{kA} \right) + \left(\frac{L}{2kA} \right) = \frac{3kA}{L}$; $R_2 = \frac{L}{3kA}$

Now, $\Delta Q_1 = \Delta Q_2$, $\frac{\Delta T}{R_1} t_1 = \frac{\Delta T}{R_2} t_2 \Rightarrow t_2 = \frac{R_2}{R_1} t_1 = 2s$

26.(B) Heat generated in device in 3 h

$$= \text{time} \times \text{power} = 3 \times 3600 \times 3 \times 10^3 = 324 \times 10^5 J$$

Heat used to change the temperature of water

$$= ms\Delta\theta = 120 \times 1 \times 4.2 \times 10^3 \times 20 J$$

Heat absorbed by coolant

$$= Pt = 324 \times 10^5 - 120 \times 1 \times 4.2 \times 10^3 \times 20 J$$

$$Pt = (324 - 100.8) \times 10^5 J, \quad P = \frac{223.2 \times 10^5}{3 \times 3600} = 2067 W$$

27.(A) Rate of the flow from Q to P $\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$

$$\text{Rate of heat flow S to R } \frac{dQ}{dt} = \frac{KA(400-T)}{1}$$

At steady state rate of heat flow is same :

$$\therefore \frac{2KA(T-10)}{1} = KA(400-T) \quad \text{or} \quad 2T-20 = 400-T$$

$$\text{or} \quad 3T = 420 \quad \therefore \quad T = 140^\circ C$$

Temperature of junction is $140^\circ C$

Temperature at a distance x from end P is $T_x = (130x + 10^\circ)$

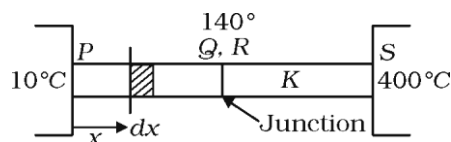
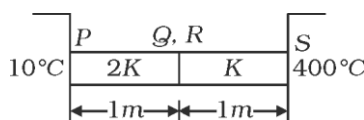
$$\left[\text{Use } \frac{kA}{1}(140-10) = \frac{kA}{x}(T_x-10) \right]$$

Change in length dx is suppose dy

Then, $dy = \alpha dx(T_x - 10)$

$$\int_0^{\Delta y} dy = \int_0^1 \alpha dx(130x + 10 - 10)$$

$$\Delta y = \left[\frac{\alpha x^2}{2} \times 130 \right]_0^1, \quad \Delta y = 1.2 \times 10^{-5} \times 65, \quad \Delta y = 78.0 \times 10^{-5} m = 0.78 mm$$



28.(C) Force in both the wires will be same.

$$\Delta x = \frac{F\ell}{Ay} = \frac{F\ell}{\pi r^2 y}$$

$$\Rightarrow \frac{\Delta x_1}{\Delta x_2} = \frac{\ell_1}{r_1^2} \times \frac{r_2^2}{\ell_2} = \frac{\ell_1}{\ell_2} \cdot \left(\frac{r_2}{r_1} \right)^2 = \left(\frac{2L}{L} \right) \left(\frac{R}{2R} \right)^2 = \frac{1}{2}$$

$$\frac{\Delta x_2}{\Delta x_1} = 2 \quad (\text{ratio of elongation of thin to thick wire})$$

29.(A) Rate of radiation energy lost by the sphere = rate of radiation energy incident

$$= \sigma(4\pi r^2)[T^4 - (300)^4] = 912 \times \pi r^2$$

$$\sigma \times T^4 = \sigma(300)^4 + \frac{912}{4}$$

$$T^4 = (300)^4 + \frac{912}{(4 \times 5.7 \times 10^{-8})} = (300)^4 + \frac{912}{22.8} \times 10^8$$

$$= (300)^4 + (40 \times 10^8) = (81 + 40) \times 10^8$$

$$= 121 \times 10^8 \quad \therefore \quad T = 330 K$$

30.(B) $T(t) = T_0(1 + \beta t^{1/4}) \Rightarrow \left[\frac{T(t) - T_0}{\beta T_0} \right]^4 = t$

Differentiating w.r.t. time

$$\frac{1}{\beta^4 T_0^4} 4[T(t) - T_0]^3 \frac{dT(t)}{dt} = 1 \quad \dots (i)$$

Also $P = \frac{dQ}{dt} = \frac{CdT}{dt} \Rightarrow \frac{dT}{dt} = \frac{P}{C}$ (C – heat capacity)

Putting in (i) : $C = \frac{4P[T(t) - T_0]^3}{\beta^4 T_0^4}$

31.(BD) The expression of radius of curvature R is $R = \frac{d}{(\alpha_1 - \alpha_2)\Delta t}$

Thus, $R \propto \frac{1}{\Delta T}$ and $R \propto \frac{1}{|\alpha_B - \alpha_C|}$.

32.(D) Since the sun rays fall on the black body, it will absorb radiations and since its temperature is constant, it will emit radiations. The temperature will remain same only when energy emitted is equal to energy absorbed.

33.(ACD) (A) At steady state, heat flow through A and E are same. So option (A) is correct and option (B) is incorrect.

(C) $\Delta T = H \times R$, where H is heat current; $R = \frac{L}{kA}$

'H' is same for A and E but R is smallest for E. So temperature difference across slab E is smallest. So option (C) is correct.

(D) $H_B = \frac{\Delta T}{R_B}$, $H_C = \frac{\Delta T}{R_C}$ and $H_D = \frac{\Delta T}{R_D}$, From this option if, $H_C = H_B + H_D$

$$\Rightarrow \frac{1}{R_C} = \frac{1}{R_B} + \frac{1}{R_D} \Rightarrow \frac{4K(2Lb)}{4L} = \frac{3K(Lb)}{4L} + \frac{5K(Lb)}{4L}$$

which is true. Here b is the width of slab. Hence option (D) is also correct.

34.(BCD) $Q = mC\Delta T \Rightarrow \frac{dQ}{dt} = mc \frac{dT}{dt}$, $R = \text{rate of absorption of heat} = \frac{dQ}{dt} \propto C$

(A) in 0 – 100 K C increase, so R increase but not linearly

(B) $\Delta Q = mC\Delta T$ as C is more in (400 K – 500 K) than (0 – 100 K) so heat is increasing

(C) C remains constant so there no change in R from (400 K – 500 K)

(D) C is increases so R increases in range (200 K – 300 K)

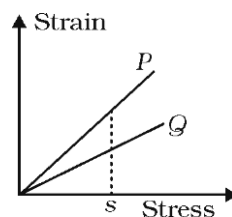
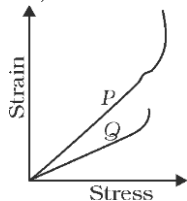
35.(AB) $Y = \frac{\text{stress}}{\text{strain}}$ or $Y \propto \frac{1}{\text{strain}}$

(for same stress say σ) $(\text{strain})_Q < (\text{strain})_P$

So, P is more ductile than Q.

Further, from the given figure we can also see that breaking stress of P is more than Q.

So, it has more tensile strength.



36.(D) Assumption $\epsilon = 1$ [black body radiation]

$$\text{Power } P = \sigma A(T^4 - T_0^4)$$

$$(C) \quad P_{rad} = \sigma AT^4 = \sigma \cdot 1 \cdot (T_0 + 10)^4 = \sigma \cdot T_0^4 \left(1 + \frac{10}{T_0}\right)^4 \quad [T_0 = 300K \text{ given}]$$

$$\text{Using binomial theorem, } P_{rad} = \sigma \cdot (300)^4 \cdot \left(1 + \frac{40}{300}\right) \approx 460 \times \frac{17}{15} \approx 520J$$

$$P_{net} = 520 - 460 \approx 60W \quad \Rightarrow \quad \text{Energy radiated in } 1 \text{ s} = 60 \text{ J}$$

$$(B) \quad P = \sigma A(T^4 - T_0^4)$$

$$dP = \sigma A(0 - 4T_0^3 \cdot dT) \quad \text{and} \quad dT = -\Delta T_0 \quad \Rightarrow \quad dP = 4\sigma AT_0^3 \Delta T_0$$

(D) If surface area decreases, then energy radiation also decreases.

NOTE While giving answer (B) and (C) it is assumed that energy radiated refers the net radiation. If energy radiated is taken as only emission, then (B) and (C) will not be included in answer.

37.(15.24) Power produced by heater = rate of heat flow through window

$$\therefore \quad \frac{V^2}{R} = \frac{\text{Temperature difference}}{\text{Thermal resistance}} = \frac{20 - \theta}{(l / KA)} \quad \therefore \quad \theta = 20 - \frac{V^2 l}{KAR}$$

$$\text{Substituting the values we have, } \theta = 20 - \frac{(200)^2 (0.2 \times 10^{-2})}{(0.2 \times 4.2)(1)(20)} = 15.24^\circ\text{C}$$

38.(12.96) 75% heat is retained by bullet

$$\frac{3}{4} \left[\frac{1}{2} m v^2 \right] = m s \Delta \theta + m L \quad \text{or} \quad v = \sqrt{\frac{(8s\Delta\theta + 8L)}{3}}$$

$$\text{Substituting the values, we have } v = \sqrt{\frac{(8 \times 0.03 \times 4.2 \times 300) + (8 \times 6 \times 4.2)}{3}} = 12.96 \text{ m/s}$$

39.(9091) Let R_1 , R_2 and R_3 be the thermal resistances of wood, cement and brick. All the resistances are in series. Hence,

$$R = R_1 + R_2 + R_3$$

$$= \frac{2.5 \times 10^{-2}}{0.125 \times 137} + \frac{1.0 \times 10^{-2}}{1.5 \times 137} + \frac{25 \times 10^{-2}}{1.0 \times 137} \quad \left(\text{as } R = \frac{l}{KA} \right)$$

$$= 0.33 \times 10^{-2} ^\circ\text{C} / \text{W}$$

$$\therefore \quad \text{Rate of heat transfer, } \frac{dQ}{dt} = \frac{\text{temperature difference}}{\text{thermal resistance}} = \frac{30}{0.33 \times 10^{-2}} \approx 9091 \text{ W}$$

$\therefore$ Power of heater should be 9091 W.

40. 166.36 sec.

For a change in temperature $d\theta$ of disc of mass M , heat given $dQ = MSd\theta$ of where S is specific heat.

$$\text{Thus the rate of heat supplied } \frac{dQ}{dt} = MS \frac{d\theta}{dt}$$

Also, for the cylindrical block of cross-sectional area A , length l , maintained at one end at temperature of θ_2 . The rate of heat flow for the other end at temperature θ is

$$\frac{dQ}{dt} = KA \frac{(\theta_2 - \theta)}{l} \quad \text{or} \quad \frac{d\theta}{\theta_2 - \theta} = \frac{KA}{MSl} dt$$

Integrating the equation

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{(\theta_2 - \theta)} = \frac{KA}{MSI} \int_{t_1}^{t_2} dt$$

$$t_2 - t_1 = \frac{MSI}{KA} \ln \left(\frac{\theta_2 - \theta_1}{\theta_2 - \theta_1} \right)$$

$$\text{Gives } (t_2 - t_1) = \frac{MSI}{KA} \ln \left[\frac{400 - 300}{400 - 350} \right] = 240 \ln 2 = 166.36 \text{ sec.}$$

41.(12) Let m be the mass of the steam required to raise the temperature of 100 g of water from 24°C to 90°C .

Heat lost by steam = Heat gained by water

$$\therefore m(L + s\Delta\theta_1) = 100s\Delta\theta_2 \quad \text{or} \quad m = \frac{(100)(s)(\Delta\theta_2)}{L + s(\Delta\theta_1)}$$

Here, s = specific heat of water = $1 \text{ cal/g}^\circ\text{C}$,

L = latent heat of vaporisation = 540 cal/g .

$$\Delta\theta_1 = (100 - 90) = 10^\circ\text{C} \quad \text{and} \quad \Delta\theta_2 = (90 - 24) = 66^\circ\text{C}$$

$$\text{Substituting the values, we have } m = \frac{(100)(1)(66)}{(540) + (1)(10)} = 12g \Rightarrow \therefore m = 12g$$

$$\mathbf{42.(60)} \quad \frac{K_A A}{L}(100 - T) = \frac{K_B A}{L}(T - 0)$$

$$\Rightarrow 300(100 - T) = 200(T) \Rightarrow 30000 = 500T \Rightarrow T = 60^\circ\text{C}$$

43. (i) 40 (ii) 0.1

(i) The change in length due to decrease in temperature,

$$\Delta l_1 = L\alpha\Delta\theta = (0.5)(10^{-5})(0 - 100)$$

$$\Delta l_1 = -0.5 \times 10^{-3} \text{ m} \quad \dots \text{ (i)}$$

Negative sign implies that length is decreasing. Now, let M be the mass attached to the lower end. Then, change in length due to suspension of load is

$$\Delta l_2 = \frac{(Mg)L}{AY} = \frac{(M)(10)(0.5)}{(4 \times 10^{-6})(10^{11})} \quad [\text{Actually } L = 0.5 - (0.5 \times 10^{-3}) \approx 0.5m]$$

$$\Delta l_2 = (1.25 \times 10^{-5})M \quad \dots \text{ (ii)}$$

$$\Delta l_1 + \Delta l_2 = 0 \quad \text{or} \quad (1.25 \times 10^{-5})M = (0.5 \times 10^{-3}) \therefore M = \left(\frac{0.5 \times 10^{-3}}{1.25 \times 10^{-5}} \right) \text{ kg} \quad \text{or}$$

$$M = 40 \text{ kg}$$

(ii) Energy stored,

At 0°C the natural length of the wire is less than its actual length; but since a mass is attached at its lower end, an elastic potential energy is stored in it. This is given by

$$U = \frac{1}{2} \left(\frac{AY}{L} \right) (\Delta l)^2 \quad \dots \text{ (iii)}$$

$$\text{Here, } \Delta l = |\Delta l_1| = \Delta l_2 = 0.5 \times 10^{-3} m$$

$$\text{Substituting the values, } U = \frac{1}{2} \left(\frac{4 \times 10^{-6} \times 10^{11}}{0.5} \right) (0.5 \times 10^{-3})^2 = 0.1 \text{ J}$$

44.(4.57) Given,

Length of the wire, $l = 5m$

Radius of the wire, $r = 2 \times 10^{-3}m$

Density of wire, $\rho = 7860 \text{ kg/m}^3$

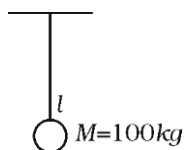
Young's modulus, $Y = 2.1 \times 10^{11} \text{ N/m}^2$

And specific heat, $s = 420 \text{ J/kg-K}$

Mass of wire, $m = (\text{density}) (\text{volume})$

$$= (\rho)(\pi r^2 l)$$

$$= (7860)(\pi)(2 \times 10^{-3})^2(5) \text{ kg} = 0.494 \text{ kg}$$



Elastic potential energy stored in the wire, $U = \frac{1}{2} (\text{stress}) (\text{strain}) (\text{volume})$

$$\left[\because \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \times \text{stress} \times \text{strain} \right] \quad \text{or} \quad U = \frac{1}{2} \left(\frac{Mg}{\pi r^2} \right) \left(\frac{\Delta l}{l} \right) (\pi r^2 l)$$

$$= \frac{1}{2} (Mg) \cdot \Delta l \quad \left(\Delta l = \frac{Fl}{AY} \right) = \frac{1}{2} (Mg) \frac{(Mgl)}{(\pi r^2)Y} = \frac{1}{2} \frac{M^2 g^2 l}{\pi r^2 Y}$$

Substituting the values, we have

$$U = \frac{1}{2} \frac{(100)^2 (10)^2 (5)}{(3.14)(2 \times 10^{-3})^2 (2.1 \times 10^{11})} \text{ J} = 0.9478 \text{ J}$$

When the bob gets snapped, this energy is utilized in raising the temperature of the wire.

So, $U = ms\Delta\theta$

$$\therefore \Delta\theta = \frac{U}{ms} = \frac{0.9478}{0.494(420)} ^\circ\text{C or K}, \quad \Delta\theta = 4.568 \times 10^{-3} ^\circ\text{C}$$

45.(2) When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same.

Upthrust = Weight. Therefore, upthrust should not change

$$F = F'$$

$$\therefore V_i \rho_L g = V_i' \rho_L' g \quad (V_i = \text{volume immersed})$$

$$\therefore (Ah_1)(\rho_L)(g) = A(1 + 2\alpha_s \Delta T)(h_1) \left(\frac{\rho_L}{1 + \gamma_l \Delta T} \right) g$$

Solving this equation, we get $\gamma_l = 2\alpha_s \therefore n = 2$

46. (i) $T_{\text{final}} = 70^\circ\text{C}$, **(ii)** 0.05 J **(iii)** 19999.95 J

47.(273) 0.05 kg steam at 373 K $\xrightarrow{Q_1}$ 0.05 kg water at 373 K

0.05 kg water at 373 K $\xrightarrow{Q_2}$ 0.05 kg water at 237 K

0.45 kg ice at 253 K $\xrightarrow{Q_3}$ 0.45 kg ice at 273 K

0.45 kg ice at 273 K $\xrightarrow{Q_4}$ 0.45 kg water at 273 K

$$Q_1 = (50)(540) = 27000 \text{ cal} = 27 \text{ kcal}$$

;

$$Q_2 = (50)(1)(100) = 5000 \text{ cal} = 5 \text{ kcal}$$

$$Q_3 = (450)(0.5)(20) = 4500 \text{ cal} = 4.5 \text{ kcal}$$

;

$$Q_4 = (450)(80) = 36000 \text{ cal} = 36 \text{ kcal}$$

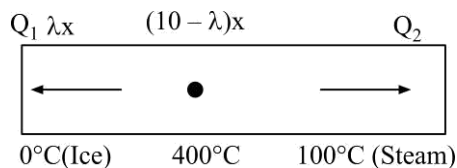
Now, since $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$ ice will come to 273 K from 253 K, but whole ice will not melt. Therefore, temperature of the mixture is 273 K.

48.(9) $\frac{dm_{ice}}{dt} = \frac{dm_{vapour}}{dt}$

$$Q_1 = \frac{KA400t}{\lambda x} = m \times 80$$

$$Q_2 = \frac{KA(400 - 100)t}{(10 - \lambda)x} = m \times 540$$

Dividing both, $\lambda = 9$.



49.(9) $\lambda_m T = \text{constant}$. $\lambda_A T_A = \lambda_B T_B$

Now rate of total energy radiated $\propto AT^4 \Rightarrow \frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \times \left(\frac{R_A}{R_B}\right)^2 = (3)^4 \times \left(\frac{6}{18}\right)^2 = 9$

50.(8) Using conservation of energy.

$$420 = (m \times 10^{-3})(2100)(5) + (1 \times 10^{-3})(3.36 \times 10^5)$$

Solving this equation, we get $m = 8 \text{ g}$ $\therefore$ The correct answer is 8.

51.(3) $\frac{F}{A} = Y \frac{\Delta L}{L} \Rightarrow \frac{mg}{A} = Y(\alpha \Delta \theta)$

$$m = \frac{Ay\alpha(\Delta\theta)}{g} = \frac{\pi r^2 y \alpha (\Delta\theta)}{g} = \frac{\pi (10^{-3})^2 \times 10^{11} \times 10^{-5} \times 10}{10} = \pi \approx 3 \text{ kg}.$$

52.(2) Let A be the surface area of smaller body then,

$$P_A = \sigma \cdot (16 \times 10^4) AT_A^4 \Rightarrow P_B = \sigma AT_B^4$$

Now, $\frac{16 \times 10^4 T_A^4}{T_B^4} = 10^4 \Rightarrow \left(\frac{T_A}{T_B}\right)^4 = \frac{1}{16} \Rightarrow \frac{T_A}{T_B} = \frac{1}{2}$

By Wein's law, $\lambda T = \text{const}$ $\therefore$ ratio $\frac{\lambda_A}{\lambda_B} = 2$.

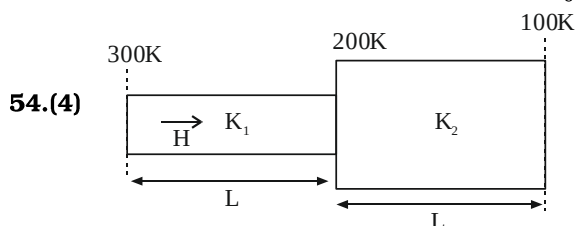
53.(9) $\log_2 \frac{P_1}{P_0} = 1$, Therefore, $\frac{P_1}{P_0} = 2$

According to Stefan's law, $P \propto T^2$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{2767 + 273}{487 + 273}\right)^4 = 4^4$$

$$\frac{P_2}{P_1} = \frac{P_2}{2P_0} = 4^4, \quad \frac{P_2}{P_0} = 2 \times 4^4, \quad \log_2 \frac{P_2}{P_0} = \log_2 [2 \times 4^4]$$

$$= \log_2 2 + \log_2 4^4, \quad \log_2 \frac{P_2}{P_0} = 1 + \log_2 2^8 = 9$$



$$H = \frac{K_1 \pi (r_1)^2 (300 - 200)}{L} = \frac{K_2 \pi (r_2)^2 (200 - 100)}{L} \Rightarrow \frac{K_1}{K_2} = \left(\frac{r_2}{r_1}\right)^2 = 4$$

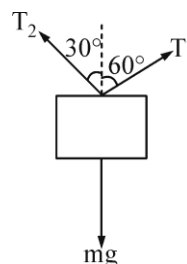
55.(2.00) $T_1 \sin 60^\circ = T_2 \sin 30^\circ \Rightarrow T_2 = \sqrt{3} T_1$

$$T_1 \cos 60^\circ + T_2 \cos 30^\circ = mg$$

$$\frac{T_1}{2} + \sqrt{3} T_1 \frac{\sqrt{3}}{2} = 100 \Rightarrow 2T_1 = 100 \Rightarrow T_1 = 50N$$

$$T_2 = \sqrt{3} T_1 = 50\sqrt{3}N$$

$$\Delta \ell = \frac{T \ell}{AY} \quad \therefore \quad \frac{\Delta \ell c}{\Delta \ell s} = \frac{T_1}{T_2} \frac{\ell_1}{\ell_2} \frac{Y_2}{Y_1} = \left(\frac{1}{\sqrt{3}} \right) \left(\frac{\sqrt{3}}{1} \right) (2) = 2$$



56.(270.00) $5(s) (80 - 30) + 5L = C(110 - 80)$, where C is heat capacity of calorimeter

$$\Rightarrow 250s + 5L = 30C, \quad 80(s) (50 - 30) = C(80 - 50)$$

$$\Rightarrow 1600s = 30c \quad \therefore \quad 250s + 5L = 1600s \Rightarrow 5L = 1350s \Rightarrow \frac{L}{s} = 270$$

57. (Partly solid & Partly liquid)

$O \rightarrow A$: Material is solid & its Temperature is increasing

$A \rightarrow B$: Phase transition takes place so material is partly solid & partly liquid.

58. $\left(\frac{1}{2} \left(\frac{YA}{L} \right) x^2 \right)$

$$W = \frac{1}{2} Kx^2, \quad \text{Here, } K = \frac{YA}{L} \quad \therefore \quad W = \frac{1}{2} \left(\frac{YA}{L} \right) x^2$$

59. $\left(\frac{Mg}{3Ak} \right)$

$$\Delta p = \frac{Mg}{A} \Rightarrow \left| \frac{\Delta V}{V} \right| = \frac{\Delta p}{k} = \frac{Mg}{Ak}, \quad \text{Now as, } V = \frac{4}{3} \pi R^3 \text{ or } V \propto R^3$$

$$\therefore \quad \frac{\Delta V}{V} = 3 \left(\frac{\Delta R}{R} \right) \quad \text{or} \quad \frac{\Delta R}{R} = \frac{1}{3} \left(\frac{\Delta V}{V} \right) \quad \dots (i)$$

$$\therefore \quad \text{From equation (i)} \quad \frac{\Delta R}{R} = \frac{Mg}{3Ak}$$

60. (0) Heat liberated when 300 g water at 25°C goes to water at 0°C : $Q = ms\Delta\theta = (300)(1)(25) = 7500 \text{ cal}$

$$\text{From } Q = mL, \text{ this much heat can melt mass of ice given by } m = \frac{Q}{L} = \frac{7500}{80} = 93.75g$$

i.e. whole ice will not melt.

Hence, the mixture will be at 0°C .

$$\text{Mass of water in mixture} = 300 + 93.75, = 393.75g \text{ and}$$

$$\text{Mass of ice in mixture} = 100 - 93.75 = 6.25g$$

61. $\left(\frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T} \right)$

The condition of floating is,

Weight = Upthrust

$$V \rho_1 g = V_f \rho_2 g$$

(ρ_1 = density of metal, ρ_2 = density of mercury)

$$\therefore \quad \frac{V_f}{V} = \frac{\rho_1}{\rho_2}$$

= fraction of volume of metal submerged in mercury

= x (say)

Now, when the temperature is increased by ΔT .

$$\rho'_1 = \frac{\rho_1}{1 + \gamma_1 \Delta T} \text{ and } \rho'_2 = \frac{\rho_2}{1 + \gamma_2 \Delta T} \quad \therefore \quad x' = \left(\frac{\rho_1}{1 + \gamma_1 \Delta T} \right) \left(\frac{1 + \gamma_2 \Delta T}{\rho_2} \right)$$

$$= \frac{\rho_1}{\rho_2} \left(\frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T} \right) \quad \therefore \quad x' = x \left(\frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T} \right) \quad \therefore \quad \frac{x'}{x} = \frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T}$$

62.(1.71) Area of sphere, $A = 4\pi r^2$

Mass of sphere, $m = \left(\frac{4}{3} \pi \rho r^3 \right)$, Now, energy radiated per second = $(\sigma A T^4)$

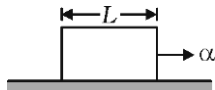
$$\therefore \quad mc \left(-\frac{dT}{dt} \right) = \sigma A T^4 \quad \text{or} \quad \int_0^t dt = \left(\frac{-mc}{\sigma A} \right) \int_{200}^{100} T^{-4} dT$$

$$\text{or} \quad t = \frac{mc}{3\sigma A} \left[\frac{1}{(100)^3} - \frac{1}{(200)^3} \right] = \frac{7mc}{3 \times 8\sigma A} \times 10^{-6} = \frac{7mc \times 10^{-6}}{24\sigma A}$$

$$\text{Substituting the values, } t = \frac{\left(7 \times \frac{4}{3} \pi \rho r^3 \right) (c) \times 10^{-6}}{3 \times 8 \times 5.67 \times 10^{-8} \times 4\pi r^2} = 1.71 \rho r c$$

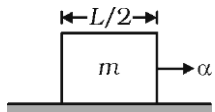
63. $\left(\frac{\rho L \alpha}{2} \right)$

Let A be area of cross-section of the rod.



Mass of half rod, $m = \text{volume} \times \text{density} = \left(\frac{L}{2} \cdot A \right) \rho$

FBD of rod at mid-point



$$\therefore \quad T = m\alpha = \left(\frac{L}{2} A \rho \alpha \right) \quad \therefore \quad \text{Stress} = \frac{T}{A} = \frac{1}{2} \rho \alpha L$$

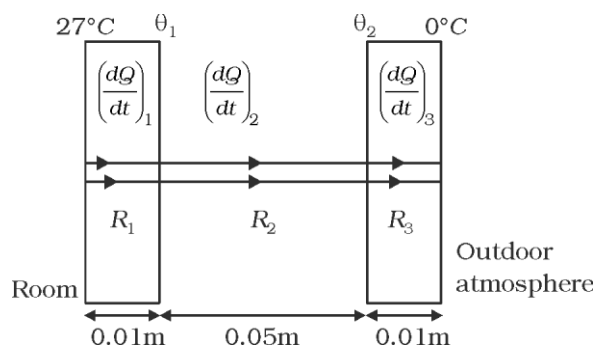
64. (i) 41.5 (ii) 26.48, 0.52

Let θ_1 and θ_2 be the temperatures of the two interfaces as shown in figure

Thermal resistance, $R = \frac{l}{KA}$

$$\therefore \quad R_1 = R_3 = \frac{(0.01)}{(0.8)(1)} = 0.0125 \text{ K/W or } ^\circ\text{C/W}$$

$$\text{and} \quad R_2 = \frac{(0.05)}{(0.08)(1)} = 0.625 ^\circ\text{C/W}$$



Now, the rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be equal

from all the three sections and since rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{Thermal resistance}} \quad \text{and} \quad \left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 = \left(\frac{dQ}{dt}\right)_3$$

Therefore,
$$\frac{27 - \theta_1}{0.0125} = \frac{\theta_1 - \theta_2}{0.625} = \frac{\theta_2 - 0}{0.0125}$$

Solving this equation, we get $\theta_1 = 26.48^\circ\text{C}$ and $\theta_2 = 0.52^\circ\text{C}$

and
$$\frac{dQ}{dt} = \frac{27 - \theta_1}{0.0125} \Rightarrow \frac{dQ}{dt} = \frac{(27 - 26.48)}{0.0125} = 41.5\text{W}$$

65.(5.5) Heat required to melt the block $= (0.28\text{kg})(3.3 \times 10^5\text{J/kg}) = 9.24 \times 10^4\text{J}$

Heat received per second $= (1400\text{J/m}^2)(0.2\text{m}^2) = 280\text{J}$

$\therefore$ Time taken (in minutes) $= \frac{9.24 \times 10^4}{280 \times 60} = 5.5$

66.
$$\left(300 + 12.5e^{\frac{-2KA t_1}{CL}} \right)$$

In the first part of the question ($t \leq t_1$)

At $t = 0$, $T_X = T_0 = 400\text{K}$ and at $t = t_1$, $T_X = T_1 = 350\text{K}$

Temperature of atmosphere, $T_A = 300\text{K}$ (constant)

This cools down according to Newton's law of cooling.

Therefore, rate of cooling $\propto$ temperature difference.

$$\therefore \left(-\frac{dT}{dt}\right) = k(T - T_A) \Rightarrow \frac{dT}{T - T_A} = -k dt \Rightarrow \int_{T_0}^{T_1} \frac{dT}{T - T_A} = -k \int_0^{t_1} dt$$

$$\Rightarrow \ln\left(\frac{T_1 - T_A}{T_0 - T_A}\right) = -kt_1 \Rightarrow kt_1 = -\ln\left(\frac{350 - 300}{400 - 300}\right)$$

$$\Rightarrow kt_1 = \ln(2) \quad \dots (i)$$

In the second part ($t > t_1$), body X cools by radiation

(according to Newton's law) as well as by conduction.

Therefore, rate of cooling = (cooling by radiation) + (cooling by conduction)

In conduction,
$$\frac{dQ}{dt} = \frac{KA(T - T_A)}{L} = C\left(-\frac{dT}{dt}\right) \therefore \left(-\frac{dT}{dt}\right) = \frac{KA}{LC}(T - T_A)$$

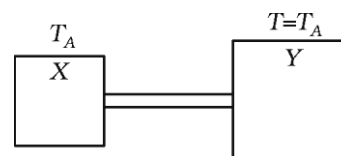
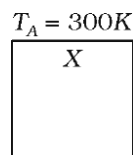
where, C = heat capacity of body X

$$\therefore \left(-\frac{dT}{dt}\right) = k(T - T_A) + \frac{KA}{CL}(T - T_A) \quad \dots (ii)$$

$$\left(-\frac{dT}{dt}\right) = \left(k + \frac{KA}{CL}\right)(T - T_A) \quad \dots (iii)$$

Let at $t = 3t_1$, temperature of x becomes T_2

Then from Eq. (iii)



$$\int_{T_1}^{T_2} \frac{dT}{T - T_A} = - \left(k + \frac{KA}{KC} \right) \int_{t_1}^{3t_1} dt$$

$$\ln \left(\frac{T_2 - T_A}{T_1 - T_A} \right) = - \left(k + \frac{KA}{LC} \right) (2t_1) = - \left(2kt_1 + \frac{2KA}{LC} t_1 \right)$$

$$\text{or } \ln \left(\frac{T_2 - 300}{350 - 300} \right) = -2 \ln(2) - \frac{2KA t_1}{LC}; \quad kt_1 = \ln(2) \text{ from Eq. (i).}$$

$$\text{This equation gives } T_2 = \left(300 + 12.5e^{\frac{-2KA t_1}{LC}} \right) K$$

67. $\left(\frac{K}{4e\sigma L T_s^3 + K} \right)$

Rate of heat conduction through rod = rate of the heat lost from right end of the rod.

$$\therefore \frac{KA(T_1 - T_2)}{L} = eA\sigma(T_2^4 - T_s^4) \quad \dots (i)$$

$$\text{Given that } T_2 = T_s + \Delta T$$

$$\therefore T_2^4 = (T_s + \Delta T)^4 = T_s^4 \left(1 + \frac{\Delta T}{T_s} \right)^4$$

Using binomial expansion, we have

$$T_2^4 = T_s^4 \left(1 + 4 \frac{\Delta T}{T_s} \right) \quad (\text{as } \Delta T \ll T_s)$$

$$\therefore T_2^4 - T_s^4 = 4(\Delta T)(T_s^3)$$

$$\text{Substituting in Eq. (i), we have } \frac{K(T_1 - T_s - \Delta T)}{L} = 4e\sigma T_s^3 \Delta T$$

$$\text{or } \frac{K(T_1 - T_s)}{L} = \left(4e\sigma T_s^3 + \frac{K}{L} \right) \Delta T \quad \therefore \Delta T = \frac{K(T_1 - T_s)}{(4e\sigma L T_s^3 + K)}$$

$$\text{Comparing with the given relation, proportionality constant} = \frac{K}{4e\sigma L T_s^3 + K}$$

TRUE / FALSE TYPE

68. **False**

When tube is heated the density of water at A will decrease, hence the water will rise up or it will circulate in clockwise direction.

69. **False**

$$h = \frac{P_0}{\rho g}$$

70. **False**

Energy radiated per second $\propto AT^4 \propto (R^2)T^4$

$$\frac{E_1}{E_2} = \left(\frac{1}{4} \right)^2 \left(\frac{4000}{2000} \right)^4 = 1 \quad \text{or} \quad E_1 = E_2$$

71. **A-S; B-Q; C-P, Q; D-Q, R**

Self-explanatory